

Gluon self-energy effect on CFL gap equation

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I investigate the effects of the gluon self-energy in the color-flavor-locked phase of color superconductivity on the value of the gap. The light plasmon modes, which appear for energies smaller than twice the gap energy, provide the largest contribution. They modify the subsubleading term in the solution of the gap equation.

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I. INTRODUCTION

At very high densities, $\mu \gg \Lambda_{QCD}$, single-gluon exchange is the dominant interaction between quarks. This interaction is attractive in the color-antitriplet channel. Therefore, sufficiently cold and dense quark matter is a color superconductor [1]. At such densities, asymptotic freedom [2] implies that the strong coupling constant, g , is much smaller than unity, $g(\mu) \ll 1$. Hence, in this limit, the calculations of the color-superconducting gap parameter ϕ can be controlled.

In order to compute the gap parameter one has to solve the gap equation, which is in general a function of the fermionic quasiparticles four-momentum $K_\mu \equiv (k_0, \mathbf{k})$. For an ordinary superconductor, in the weak coupling limit, $p_D \sim g\mu \ll \mu$, where p_D is the Debye momentum, the the solution of the gap equation is

$$\phi = b_{BCS} \mu \exp \left(-\frac{c_{BCS}}{g^2} \right). \quad (1)$$

Here, $b_{BCS} \equiv 2\omega_D/\mu$ is a dimensionless constant and ω_D is the Debye frequency. To derive Eq.(1) it is assumed that the attractive phonon exchange interaction between electrons is local [3, 4]. The locality for the exchanged particles, however, is not a crucial condition. One can generalize the solution to nonlocal interactions of finite range, for instance, to the massive scalar boson exchange [5]. If we assume that fermions are massless and the boson mass is generated by in-medium effects, $M_B \simeq g\mu$, the energy scales in the weak coupling limit are $\phi \ll M_b \ll \mu$. The solution of the gap equation is then

$$\phi \simeq b_B \mu \exp \left(-\frac{c_B}{g^2 \ln(2\mu/M_B)} \right). \quad (2)$$

The main difference of this solution with Eq.(1) is that the gap becomes larger, i.e. g^2 is replaced by $g^2 \ln(2\mu/M_B) \sim g^2 \ln(1/g) \gg g^2$.

In QCD gluon exchange is a nonlocal interaction. In the vacuum gluons are massless; hence, the interaction is of infinite range. In the dense medium, however, the electric and the nonstatic magnetic gluon exchanges are screened. In the latter case, the behavior of the gluons is very similar to the scalar bosons of the previous example with a mass

$$m_g^2 = N_f \frac{g^2 \mu^2}{6\pi^2} + \left(N_c + \frac{N_f}{2} \right) \frac{g^2 T^2}{9}, \quad (3)$$

where N_f and N_c are the number of massless quark flavors and quark colors participating in the pairing, respectively. T is the temperature. Notice that, the exchange of the almost static magnetic gluons is still unscreened and thus has infinite range [6]. In the weak coupling limit, all unscreened gluons contribute to leading order to the gap equation, whereas all screened gluons contribute to subleading order. All other contributions construct the subsubleading term [7].

In comparison to the BCS theory, where the maximum phonon momentum is the Debye momentum p_D , in QCD there is no restriction on the magnitude of the gluon momentum in the gap equation. Nevertheless, in the weak

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coupling limit, the gap function is strongly peaked at the Fermi surface $q = k_F$, and this indirectly again restricts the gluon momentum to a narrow range around the Fermi surface, $k_F - \delta \leq q \leq k_F + \delta$ [6, 8, 9]. Under this condition, the solution of the gap equation is

$$\phi = b \mu \exp\left(-\frac{c}{g}\right) [1 + O(g)], \quad (4)$$

where $b = 256\pi^4(2/g^2 N_f)^{5/2} b'_0$. The dependence of b on g arises from the gluon mass, Eq. (3). At the present approximation $b'_0 = 1$ [6, 9, 10, 11]. Including a finite, μ -dependent contribution to the quark wave function renormalization, Brown, Liu, and Ren in Ref. [12] showed that

$$b'_0 = \exp\left(-\frac{\pi^2 + 4}{8}\right) b''_0 \simeq 0.176 b''_0. \quad (5)$$

Using the standard running coupling constant in the vacuum, one can find $b''_0 = \exp[33(\pi^2 - 4)] \simeq 20$ [13]. We see that in comparison to Eqs. (1) and (2) the power of the coupling constant g in the exponent of Eq. (4) is reduced [14]. In the hard-dense-loop (HDL) limit, the constant c in Eq. (4) is calculated by Son [15, 16]:

$$c = \frac{3\pi^2}{\sqrt{2}}. \quad (6)$$

In the other works [17], the effect of the quark self-energy is taken into account. This gives rise to a more accurate value for b'_0 . Later on, the solution of the gap equation for different phases of color superconductivity were presented in several papers [18]. Recently, it is also shown that the subleading order contributions to the gap from the vertex corrections are absent for the gapped excitations [19].

In this paper, at asymptotically large densities $\phi \ll m_g \ll \mu$, I investigate the effect of the gluon self-energy on the solution of the gap in the color-flavor-locked (CFL) phase of color superconductor. It is believed that this effect is of leading or subleading order [20, 21]. These calculations are for the complete range of the gluon energy and momentum $p_0, p \ll \mu$. I consider small temperatures $T \sim \phi \ll \mu$ where the dominant contribution to the one-loop gluon self-energy comes from the quark loops, which are $\sim g^2 \mu^2$. At this limit, the contribution of the gluon and the ghost loops are suppressed, because they are proportional to $g^2 T^2$ [21].

The gluon self-energies in the CFL phase are explicitly calculated in Refs. [20, 22]. It is known that there is a light plasmon mode in the CFL phase. Therefore, the other aim of this paper is to know the effect of these unique modes on the value of the CFL gap. Similar calculations for the 2SC phase are given in Ref. [7], except, in those calculations the light plasmon mode is not studied.

This paper is organized as follows. In Sec. II, I solve the gap equation in the color-flavor-locked phase. For that, I need to know the full gluon propagators. In Sec. II A, it is shown how one can write these propagators in terms of their spectral densities. The discussions about the different limits of the gluon spectral densities are presented in II B. The form of the gap equation requires one to have knowledge about the dispersion relation of the gluons. This is explained in II C. Collecting all these materials, we become ready to estimate the effect of the gluon self-energy on the solution of the gap. Section III is divided to three subsections. Subsections III A and III B are devoted to estimate the effect of the gluon self-energy in color-superconducting phase in comparison to that in the HDL limit. In III C, I achieve my goal and find the effect of the largest contribution from the gluon self-energy on the gap function. In the end, I conclude.

II. GAP EQUATION

The gap equation for the color-superconducting condensate of the massless fermions is derived in [5]. At nonzero temperature this equation reads

$$\Phi^+ = g^2 \frac{T}{V} \sum_Q \bar{\Gamma}_a^\mu \Delta_{\mu\nu}^{ab}(K - Q) G_0^-(Q) \Phi^+(Q) G^+(Q) \Gamma_b^\nu, \quad (7)$$

where a summation over Lorentz indices μ, ν as well as adjoint color indices $a, b = 1, \dots, 8$ is implied, and in the finite-volume limit we have

$$\frac{T}{V} \sum_Q \equiv \sum_n \int \frac{d^3 \mathbf{q}}{2\pi^3}. \quad (8)$$

Here n labels the fermionic Matsubara frequencies

$$\omega_n \equiv (2n + 1)\pi T \equiv iq_0. \quad (9)$$

The full gluon propagator is related to the gluon self-energy $\Pi_{\mu\nu}^{ab}$ via

$$[\Delta^{-1}]_{\mu\nu}^{ab} = [\Delta_0^{-1}]_{\mu\nu}^{ab} - \Pi_{\mu\nu}^{ab}, \quad (10)$$

where $[\Delta_0^{-1}]_{\mu\nu}^{ab}$ is the bare gluon propagator. The propagator for the free, massless particles (upper sign) or charge-conjugate particles (lower sign),

$$[G_0^\pm(Q)]^{-1} \equiv \gamma \cdot Q \pm \gamma_0 \mu, \quad (11)$$

constitutes the propagator for the quasiparticles (upper sign) or the charge-conjugate quasiparticles (lower sign) as follows

$$[G^\pm(Q)]^{-1} \equiv [G_0^\pm(Q)]^{-1} - \Sigma^\pm(Q). \quad (12)$$

The quark self-energy arises from the interaction with the condensate

$$\Sigma^\pm(Q) \equiv \Phi^\mp G_0^\mp \Phi^\pm. \quad (13)$$

The charge-conjugate condensate Φ^- is related to the condensate Φ^+ via

$$\Phi^- \equiv \gamma_0(\Phi^+)^\dagger \gamma_0, \quad (14)$$

and the vertices in Eq.(7) are defined as

$$\Gamma_a^\mu \equiv T_a \gamma^\mu, \quad \bar{\Gamma}_a^\mu \equiv -\gamma^\mu T_a^T, \quad (15)$$

where γ^μ is the Dirac matrix and T_a the Gell-Mann matrix.

Using the following projectors

$$\mathcal{C}_{ij}^{(1)fg} \equiv \frac{1}{3} \delta_i^f \delta_j^g, \quad (16a)$$

$$\mathcal{C}_{ij}^{(2)fg} \equiv \frac{1}{2} \left(\delta^{fg} \delta_{ij} - \delta_j^f \delta_i^g \right), \quad (16b)$$

$$\mathcal{C}_{ij}^{(3)fg} \equiv \frac{1}{2} \left(\delta^{fg} \delta_{ij} + \delta_j^f \delta_i^g \right) - \frac{1}{3} \delta_i^f \delta_j^g, \quad (16c)$$

from Ref.[23], the gap matrix Eq.(7) can be written as

$$\Phi^\pm \equiv \sum_{n=1}^3 \Phi_n^\pm \quad (17)$$

where

$$\Phi_1^\pm \equiv 2(\Phi_3^\pm + 2\Phi_6^\pm), \quad (18a)$$

$$\Phi_2^\pm \equiv \Phi_3^\pm - \Phi_6^\pm, \quad (18b)$$

$$\Phi_3^\pm \equiv -\Phi_2^\pm, \quad (18c)$$

are gap matrices in the spinor space,

$$\Phi_n^+ \equiv \sum_{h=r,\ell} \sum_{e=\pm} \phi_{n,h}^e(K) \mathcal{P}_h \Lambda_{\mathbf{k}}^e, \quad (19)$$

$$\Phi_n^- \equiv \sum_{h=r,\ell} \sum_{e=\pm} [\phi_{n,h}^e(K)]^* \mathcal{P}_{-h} \Lambda_{\mathbf{k}}^{-e}. \quad (20)$$

The chirality projectors

$$\mathcal{P}_{r,\ell} \equiv \frac{1}{2}(1 \pm \gamma_5) \quad (21)$$

are defined so that $-h = \ell$ when $h = r$ and $-h = r$ when $h = \ell$. In addition

$$\Lambda_{\mathbf{k}}^{\pm} \equiv \frac{1}{2}(1 \pm \gamma_0 \boldsymbol{\gamma} \cdot \hat{\mathbf{k}}) \quad (22)$$

are energy projectors and $\phi_{n,h}^e(K)$ in Eq. (19) is the gap function for the pairing of quarks ($e = +1$) or antiquarks ($e = -1$) with chirality h .

From Eq. (18) one can write the gap matrix Eq. (7) in terms of the triplet $\Phi_{\mathbf{3}}^{\pm}$ and the sextet $\Phi_{\mathbf{6}}^{\pm}$ gaps

$$[\Phi^{\pm}]_{ij}^{fg} \equiv \Phi_{\mathbf{3}}^{\pm}(\delta_i^f \delta_j^g - \delta_j^f \delta_i^g) + \Phi_{\mathbf{6}}^{\pm}(\delta_i^f \delta_j^g + \delta_j^f \delta_i^g). \quad (23)$$

For convenience we use the following notations:

$$\Phi_{\mathbf{1}}^{\pm} \equiv \Phi_1^{\pm}, \quad \Phi_{\mathbf{8}}^{\pm} \equiv \Phi_2^{\pm} \equiv -\Phi_3^{\pm}. \quad (24)$$

Hence, similar to Eq. (17), one can write the quasiparticle propagators in terms of the projectors introduced in Eq. (16)

$$G^{\pm}(K) \equiv \sum_{n=1}^3 G_n^{\pm}, \quad (25)$$

where

$$G_n^{\pm}(K) \equiv \sum_{h=r,\ell} \sum_{e=\pm} \frac{\mathcal{P}_{\pm h} \Lambda_{\mathbf{k}}^{\pm e}}{k_0^2 - [\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e)]^2} [G_0^{\mp}(K)]^{-1}. \quad (26)$$

Here, $\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e)$ is the quasiparticles energy

$$[\epsilon_{\mathbf{k}}^e(\phi_{n,h}^e)]^2 \equiv (\mu - ek)^2 + |\phi_{n,h}^e|^2. \quad (27)$$

Using Eq. (24) and Eq. (26) we have

$$[G^{\pm}]_{ij}^{fg} \equiv [\mathbf{P}_{\mathbf{1}}]_{ij}^{fg} G_{\mathbf{1}}^{\pm} + [\mathbf{P}_{\mathbf{8}}]_{ij}^{fg} G_{\mathbf{8}}^{\pm}, \quad (28)$$

where

$$[\mathbf{P}_{\mathbf{1}}]_{ij}^{fg} \equiv \frac{1}{3} \delta_i^f \delta_j^g, \quad (29a)$$

$$[\mathbf{P}_{\mathbf{8}}]_{ij}^{fg} \equiv \delta^{fg} \delta_{ij} - \frac{1}{3} \delta_i^f \delta_j^g. \quad (29b)$$

Combining Eqs. (16)-(18), we can write a relation similar to that in Eq. (28) for the gap matrix

$$[\Phi^{\pm}]_{ij}^{fg} \equiv [\mathcal{Q}_{\mathbf{1}}]_{ij}^{fg} \Phi_{\mathbf{1}}^{\pm} + [\mathcal{Q}_{\mathbf{8}}]_{ij}^{fg} \Phi_{\mathbf{8}}^{\pm}, \quad (30)$$

but with different projectors

$$[\mathcal{Q}_{\mathbf{1}}]_{ij}^{fg} \equiv \frac{1}{3} \delta_i^f \delta_j^g, \quad (31a)$$

$$[\mathcal{Q}_{\mathbf{8}}]_{ij}^{fg} \equiv \frac{1}{3} \delta_i^f \delta_j^g - \delta_j^f \delta_i^g. \quad (31b)$$

We see that $[\mathcal{Q}_{\mathbf{1}}]_{ij}^{fg} \equiv [\mathbf{P}_{\mathbf{1}}]_{ij}^{fg}$. Furthermore, one can check that

$$[\mathcal{Q}_{\mathbf{1}}]_{ij}^{fg} \times [\mathbf{P}_{\mathbf{1}}]_{jl}^{gh} \equiv [\mathcal{Q}_{\mathbf{1}}]_{il}^{fh}, \quad (32a)$$

$$[\mathcal{Q}_{\mathbf{1}}]_{ij}^{fg} \times [\mathbf{P}_{\mathbf{8}}]_{jl}^{gh} \equiv 0, \quad (32b)$$

$$[\mathcal{Q}_{\mathbf{8}}]_{ij}^{fg} \times [\mathbf{P}_{\mathbf{1}}]_{jl}^{gh} \equiv 0, \quad (32c)$$

$$[\mathcal{Q}_{\mathbf{8}}]_{ij}^{fg} \times [\mathbf{P}_{\mathbf{8}}]_{jl}^{gh} \equiv [\mathcal{Q}_{\mathbf{8}}]_{il}^{fh}, \quad (32d)$$

together with

$$[\mathcal{Q}_1]_{ij}^{fg} \times [\mathcal{Q}_1]_{ji}^{gf} \equiv 1, \quad (33a)$$

$$[\mathcal{Q}_1]_{ij}^{fg} \times [\mathcal{Q}_8]_{ji}^{gf} \equiv 0, \quad (33b)$$

$$[\mathcal{Q}_8]_{ij}^{fg} \times [\mathcal{Q}_8]_{ji}^{gf} \equiv 8. \quad (33c)$$

Writing the gap equation, Eq. (7), in terms of its color and flavor indices

$$[\Phi^+]_{ij}^{fg}(K) = g^2 \frac{T}{V} \sum_Q \gamma^\mu (T_a^T)_{ic} \Delta_{\mu\nu}^{ab}(K-Q) [G_0^-(Q)]_{cm}^{fl} [\Phi^+(Q)]_{mk}^{ln} [G^+(Q)]_{kd}^{ng} (T_a)_{dj} \gamma^\nu \quad (34)$$

and using Eq. (32), the gap equation finds the form

$$[\Phi^+]_{ij}^{fg}(K) = g^2 \frac{T}{V} \sum_Q \gamma^\mu (T_a^T)_{ic} \Delta_{\mu\nu}^{ab}(K-Q) G_0^-(Q) \left[\Phi_1^+(Q) G_1^+(Q) [\mathcal{Q}_1]_{cd}^{fg} + \Phi_8^+(Q) G_8^+(Q) [\mathcal{Q}_8]_{cd}^{fg} \right] (T_b)_{dj} \gamma^\nu \quad (35)$$

Here, we used the fact that the massless bare quark propagator is diagonal in the color-flavor space,

$$[G_0^\pm]_{ij}^{fg} \equiv \delta^{fg} \delta_{ij} G_0^\pm. \quad (36)$$

To proceed, we need to know the form of the gluon propagators including the gluon self-energies in terms of the associated spectral densities. This is presented in the next subsection.

A. Gluon Propagators

In pure Coulomb gauge, the gluon propagators have the following forms:

$$\Delta_{00}^{ab}(K) \equiv -\delta^{ab} \frac{1}{\mathbf{k}^2 - \Pi_{00}^{cc}(K)}, \quad (37a)$$

$$\Delta_{0i}^{ab}(K) \equiv 0, \quad (37b)$$

$$\Delta_{ij}^{ab}(K) \equiv -\delta^{ab} \left(\delta_{ij} - \hat{k}_i \hat{k}_j \right) \frac{1}{K^2 - \Pi_t^{cc}(K)}, \quad (37c)$$

where the transverse component of the gluon self-energy is defined as

$$\Pi_t^{aa}(K) \equiv \frac{1}{2} \left(\delta^{ij} - \hat{k}^i \hat{k}^j \right) \Pi_{ij}^{aa}(K), \quad (38)$$

In addition, the longitudinal and the transverse components of the gluon propagator are defined as $\Delta_{00}^{ab}(K) \equiv -\delta^{ab} \Delta_{00}$ and $\Delta_{ij}^{ab}(K) \equiv -\delta^{ab} \left(\delta_{ij} - \hat{k}_i \hat{k}_j \right) \Delta_t$, respectively [21, 24]. We insert the gluon propagators in the gap equation we have obtained so far, and use Eq. (30) to write the right hand side (rhs) of Eq. (35) in terms of the singlet and the octet gaps. After some algebraic calculation and using the properties of the projectors in Eq. (33), eventually, we reach the following coupled gap equations:

$$\Phi_1^+(K) \equiv g^2 \frac{2T}{3V} \sum_Q \gamma^\mu \Delta_{\mu\nu}^{aa}(K-Q) G_0^-(Q) \Phi_8^+(Q) G_8^+(Q) \gamma^\nu, \quad (39a)$$

$$\Phi_8^+(K) \equiv g^2 \frac{T}{12V} \sum_Q \gamma^\mu \Delta_{\mu\nu}^{aa}(K-Q) G_0^-(Q) \left[\Phi_8^+(Q) G_8^+(Q) + 2 \Phi_1^+(Q) G_1^+(Q) \right] \gamma^\nu \quad (39b)$$

Utilizing Eq. (19) and Eq. (26) in Eq. (39a), we have

$$\Phi_1^+(K) \equiv g^2 \frac{2T}{3V} \sum_Q \sum_{h=r,\ell} \sum_{e=\pm} \gamma^\mu \Delta_{\mu\nu}^{aa}(K-Q) \mathcal{P}_{-h}^{-e} \frac{\phi_{8,h}^e(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^e(\phi_{8,h}^e)]^2} \gamma^\nu \quad (40)$$

and the same procedure leads to the following expression for Eq. (39b):

$$\Phi_{\mathbf{s}}^+(K) \equiv g^2 \frac{T}{12V} \sum_Q \sum_{h=r,\ell} \sum_{e=\pm} \gamma^\mu \Delta_{\mu\nu}^{aa}(K-Q) \mathcal{P}_{-h}^{-e} \left[\frac{\phi_{1,h}^e(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^e(\phi_{1,h}^e)]^2} + \frac{2\phi_{8,h}^e(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^e(\phi_{8,h}^e)]^2} \right] \gamma^\nu. \quad (41)$$

Here $\mathcal{P}_h^e \equiv \mathcal{P}_h \Lambda_{\mathbf{q}}^e$. Implementation of Eq. (19) in the left-hand side of the above equations gives

$$\begin{aligned} \phi_{1,h}^e(K) \equiv g^2 \frac{2T}{3V} \sum_Q \Delta_{\mu\nu}^{aa}(K-Q) \left\{ \frac{\phi_{8,h}^e(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^e(\phi_{8,h}^e)]^2} \text{Tr} [\mathcal{P}_h^e(\mathbf{k}) \gamma^\mu \mathcal{P}_{-h}^{-e}(\mathbf{q}) \gamma^\nu] \right. \\ \left. + \frac{\phi_{8,h}^{-e}(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^{-e}(\phi_{8,h}^{-e})]^2} \text{Tr} [\mathcal{P}_h^e(\mathbf{k}) \gamma^\mu \mathcal{P}_{-h}^e(\mathbf{q}) \gamma^\nu] \right\} \end{aligned} \quad (42)$$

where we used the properties of the chiral and the energy projectors. Also we find that

$$\begin{aligned} \phi_{8,h}^e(K) \equiv g^2 \frac{T}{12V} \sum_Q \Delta_{\mu\nu}^{aa}(K-Q) \left(\left\{ \frac{\phi_{1,h}^e(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^e(\phi_{1,h}^e)]^2} + \frac{2\phi_{8,h}^e(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^e(\phi_{8,h}^e)]^2} \right\} \text{Tr} [\mathcal{P}_h^e(\mathbf{k}) \gamma^\mu \mathcal{P}_{-h}^{-e}(\mathbf{q}) \gamma^\nu] \right. \\ \left. + \left\{ \frac{\phi_{1,h}^{-e}(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^{-e}(\phi_{1,h}^{-e})]^2} + \frac{2\phi_{8,h}^{-e}(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^{-e}(\phi_{8,h}^{-e})]^2} \right\} \text{Tr} [\mathcal{P}_h^e(\mathbf{k}) \gamma^\mu \mathcal{P}_{-h}^e(\mathbf{q}) \gamma^\nu] \right) \end{aligned} \quad (43)$$

For these parts we have benefited from Ref.[6] which presents the relations between the gap functions with different chiralities for the quasiparticles and the quasiantiparticles. To proceed, we apply the following identities:

$$\text{Tr} [\mathcal{P}_h \Lambda_{\mathbf{k}}^e \gamma_0 \mathcal{P}_{-h} \Lambda_{\mathbf{q}}^\mp \gamma_0] \equiv \frac{1 \pm \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2}, \quad (44a)$$

$$\sum_i \text{Tr} [\mathcal{P}_h \Lambda_{\mathbf{k}}^e \gamma_i \mathcal{P}_{-h} \Lambda_{\mathbf{q}}^\mp \gamma_i] \equiv -\frac{3 \mp \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} \quad (44b)$$

$$\text{Tr} [\mathcal{P}_h \Lambda_{\mathbf{k}}^e \gamma \cdot \hat{\mathbf{p}} \mathcal{P}_{-h} \Lambda_{\mathbf{q}}^\mp \gamma \cdot \hat{\mathbf{p}}] \equiv -\frac{1 \pm \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} \frac{(k \mp q)^2}{(\mathbf{k} - \mathbf{q})^2} \quad (44c)$$

Hence, Eq. (42) becomes

$$\begin{aligned} \phi_{1,h}^e(K) \equiv g^2 \frac{2T}{3V} \sum_Q \sum_{\alpha=\pm} \frac{\phi_{8,h}^{\alpha e}(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^{\alpha e}(\phi_{8,h}^{\alpha e})]^2} \left\{ \Delta_{00}(K-Q) \frac{1 + \alpha \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} \right. \\ \left. + \Delta_t(K-Q) \left[-\frac{3 - \alpha \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} + \frac{1 + \alpha \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} \frac{(k - \alpha q)^2}{(\mathbf{k} - \mathbf{q})^2} \right] \right\} \end{aligned} \quad (45)$$

The poles of $1/(q_0^2 - [\epsilon_{\mathbf{q}}^{\alpha e}(\phi_{8,h}^{\alpha e})]^2)$ give a residue $\sim 1/\epsilon_{\mathbf{q}}^{\alpha e}(\phi_{8,h}^{\alpha e})$. In the weak coupling limit, in the vicinity of the Fermi surface, $k \sim \mu$, the quasiparticle energy $\epsilon_{\mathbf{q}}^+$ is very small in comparison to the quasiantiparticle energy $\epsilon_{\mathbf{q}}^-$ [5]. Therefore, one can neglect the contribution of the quasiantiparticles in the gap equation, and, in the rhs, keep the terms with $\alpha e = +1$. In addition, in the weak coupling limit the approximations

$$\frac{1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} \simeq 1, \quad (46a)$$

$$-\frac{3 - \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} + \frac{1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}}{2} \frac{(k - q)^2}{(\mathbf{k} - \mathbf{q})^2} \simeq -1. \quad (46b)$$

are accepted. We now add and subtract the HDL gluon propagators to Eq. (45):

$$\phi_{1,h}^e(K) \equiv g^2 \frac{2T}{3V} \sum_Q \sum_{\alpha=\pm} \frac{\phi_{8,h}^{\alpha e}(Q)}{q_0^2 - [\epsilon_{\mathbf{q}}^{\alpha e}(\phi_{8,h}^{\alpha e})]^2} \left(\Delta_{00}^{\text{HDL}}(K-Q) - \Delta_t^{\text{HDL}}(K-Q) \right) + \delta\phi_{1,h}^e(K), \quad (47)$$

where $\delta\phi_{1,h}^e(K)$ is the difference between the gap equation in the color-superconducting phase with that in the HDL limit. Hence, for $\delta\phi_{1,h}^e(K) = 0$ we recover the standard gap equation with the gluon propagators in HDL approximation. In the weak coupling limit, the HDL solution up to subleading order is presented in Ref. [8]. The extra parts in Eq. (47) are the correction to the previous result.

To take the gluon self-energy effects into account, we have to write the gluon propagators in terms of their spectral densities. This is done in the following.

B. Spectral Densities

We now need to go to the spectral representation of the gluon propagators

$$\Delta_{00}(K) \equiv -\frac{1}{k^2} + \int_0^{1/T} d\tau e^{ik_0\tau} \Delta_{00}(\tau, \mathbf{k}), \quad (48a)$$

$$\Delta_t(K) \equiv \int_0^{1/T} d\tau e^{ik_0\tau} \Delta_t(\tau, \mathbf{k}), \quad (48b)$$

$$\Delta_{00,t}(\tau, \mathbf{k}) \equiv \int_0^\infty d\omega \rho_{00,t}(\omega, \mathbf{k}) \{ [1 + n_B(\omega/T)] e^{-\omega\tau} + n_b(\omega/T) e^{\omega\tau} \}, \quad (48c)$$

where $n_B(x) \equiv 1/(e^x - 1)$ is the Bose-Einstein distribution function. The term $-1/k^2$ in the electric propagator cancels the contribution of $\Delta_{00}(K)$ when $k_0 \rightarrow \infty$. This term is the same for the electric gluon propagator in a superconductor and the electric HDL propagator, because there are not any differences between these propagators for $k_0 \ll \phi$ [15, 25]. The spectral densities are defined as

$$\rho_{00,t}(p_0, \mathbf{k}) \equiv \frac{1}{\pi} \text{Im} \Delta_{00,t}(p_0 + i\eta, \mathbf{k}). \quad (49)$$

When $\text{Im} \hat{\Pi}_{00}(p_0, \mathbf{p})$ and $\text{Im} \Pi_t(p_0, \mathbf{p})$ are nonzero, the spectral densities are regular and the above equation is identical to

$$\begin{aligned} \rho_{00}(p_0, \mathbf{p}) &\equiv \frac{1}{\pi} \frac{\text{Im} \hat{\Pi}_{00}(p_0, \mathbf{p})}{\left[\mathbf{p}^2 - \text{Re} \hat{\Pi}_{00}(p_0, \mathbf{p}) \right]^2 + \left[\text{Im} \hat{\Pi}_{00}(p_0, \mathbf{p}) \right]^2}, \\ \rho_t(p_0, \mathbf{p}) &\equiv \frac{1}{\pi} \frac{\text{Im} \Pi_t(p_0, \mathbf{p})}{\left[p_0^2 - \mathbf{p}^2 - \text{Re} \Pi_t(p_0, \mathbf{p}) \right]^2 + \left[\text{Im} \Pi_t(p_0, \mathbf{p}) \right]^2}, \end{aligned} \quad (50)$$

where $\hat{\Pi}_{00}$ is the modified longitudinal gluon self-energy in the CFL phase, cf. Ref. [22]. When $\text{Im} \hat{\Pi}_{00}(p_0, \mathbf{p}) = \text{Im} \Pi_t(p_0, \mathbf{p}) = 0$, for a given momentum \mathbf{p} , the spectral densities have poles determined by

$$\left[\mathbf{p}^2 - \text{Re} \hat{\Pi}_{00}(p_0, \mathbf{p}) \right]_{p_0=\omega_{00}(\mathbf{p})} = 0, \quad (51a)$$

$$(51b)$$

for the electric gluons, and

$$\left[p_0^2 - \mathbf{p}^2 - \text{Re} \Pi_t(p_0, \mathbf{p}) \right]_{p_0=\omega_t(\mathbf{p})} = 0, \quad (51c)$$

for the magnetic gluons. The quasiparticle dispersion relation can be obtained by the solutions of $p_0 = \omega_{00,t}(\mathbf{p})$. Therefore in this case

$$\rho_{00,t}(p_0, \mathbf{p}) \equiv -Z_{00,t}(\mathbf{p}) \{ \delta[p_0 - \omega_{00,t}(\mathbf{p})] - \delta[p_0 + \omega_{00,t}(\mathbf{p})] \}, \quad (52)$$

where $Z_{00,t}(\mathbf{p})$ are the residues at the poles $p_0 = \omega_{00,t}(\mathbf{p})$,

$$Z_{00,t}(\mathbf{p}) \equiv \left(\left| \frac{\partial (\Delta_{00,t})^{-1}(p_0, \mathbf{p})}{\partial p_0} \right| \right)_{p_0=\omega_{00,t}(\mathbf{p})}^{-1}. \quad (53)$$

Now we can go back to Eq. (47) and continue the calculations. We can write

$$\frac{\phi_i(Q)}{q_0^2 - \epsilon_{\mathbf{q}}^2(\phi_i)} = - \int_0^{1/T} d\tau e^{q_0\tau} \frac{\phi_i(\epsilon_{\mathbf{q}}, \mathbf{q})}{2\epsilon_{\mathbf{q}}(\phi_i)} \left(\{1 - n_F[\epsilon_{\mathbf{q}}(\phi_i)/T]\} e^{-\epsilon_{\mathbf{q}}(\phi_i)\tau} - n_F[\epsilon_{\mathbf{q}}(\phi_i)/T] e^{\epsilon_{\mathbf{q}}(\phi_i)\tau} \right), \quad (54)$$

where $n_F(x) \equiv 1/(e^x + 1)$ is the Fermi-Dirac distribution function. Now, the Matsubara sum should be performed over the fermionic energies $q_0 = -i(2n+1)\pi T$. Neglecting the quasiantiparticle contributions and inserting Eqs. (48) and (54) in Eq. (47) we have

$$\delta\phi_{k,1} \equiv -\frac{2}{3}g^2 \int_0^\infty d\omega \int \frac{d^3q}{(2\pi)^3} (\delta\rho_{00} - \delta\rho_t) \frac{\phi_{q,8}}{2\epsilon_{\mathbf{k},8}} \left(\frac{1}{\omega + \epsilon_{\mathbf{q},8} + \epsilon_{\mathbf{k},8}} + \frac{1}{\omega + \epsilon_{\mathbf{q},8} - \epsilon_{\mathbf{k},8}} \right) \quad (55)$$

where, for simplicity, we dropped the subscript h and the superscript $+$, abbreviated $\phi_i(\epsilon_{\mathbf{q}}, \mathbf{q}) \equiv \phi_{q,i}$, and set $\epsilon_{\mathbf{q}}(\phi_{q,i}) = \epsilon_{\mathbf{q},i}$. We have also made an analytical continuation onto the quasiparticle mass shell, $q_0 \rightarrow \epsilon_{\mathbf{q}} + i\eta$. In Eq. (55)

$$\delta\rho_{00,t} \equiv \rho_{00,t} - \rho_{00,t}^{\text{HDL}}. \quad (56)$$

Making use of the fact that the spectral densities are isotropic, we can change the integral variable $\cos\theta \equiv \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}$ to p . Then, Eq. (55) reads

$$\delta\phi_{k,1} \simeq \frac{g^2}{12} \int_{\mu-\delta}^{\mu+\delta} dq \int_0^\infty d\omega [\mathcal{I}_t(\omega) - \mathcal{I}_{00}(\omega)] \frac{\phi_{q,8}}{\epsilon_{\mathbf{q},8}} \left(\frac{1}{\omega + \epsilon_{\mathbf{q},8} + \epsilon_{\mathbf{k},8}} + \frac{1}{\omega + \epsilon_{\mathbf{q},8} - \epsilon_{\mathbf{k},8}} \right) \quad (57)$$

where

$$\mathcal{I}_{00,t}(\omega) \equiv \int_0^{2\mu} dp \, p \, \delta\rho_{00,t}(\omega, p). \quad (58)$$

To derive these formulas, we have employed the approximations $k/q \simeq 1$, $|k - q| \simeq 0$, and $k + q \simeq 2\mu$. These approximations are justified because the q -integral peaks at the Fermi surface [7]. Moreover, since the gap is strongly dependent on q , the momentum integration is restricted to the Fermi surface $\mu - \delta \leq q \leq \mu + \delta$.

Note that the functions $\mathcal{I}_{00,t}(\omega)$ are dimensionless. Therefore, all quantities derived from them must be dimensionless as well, e.g. ω/m_g , ϕ/m_g , ω/μ , ϕ/μ , etc. We will make use of this property of $\mathcal{I}_{00,t}(\omega)$ in the coming sections.

The same procedure for Eq. (41) yields

$$\begin{aligned} \delta\phi_{k,8} \simeq \frac{g^2}{96} \int_{\mu-\delta}^{\mu+\delta} dq \int_0^\infty d\omega [\mathcal{I}_t(\omega) - \mathcal{I}_{00}(\omega)] & \left[\frac{\phi_{q,1}}{\epsilon_{\mathbf{q},1}} \left(\frac{1}{\omega + \epsilon_{\mathbf{q},1} + \epsilon_{\mathbf{k},1}} + \frac{1}{\omega + \epsilon_{\mathbf{q},1} - \epsilon_{\mathbf{k},1}} \right) \right. \\ & \left. + 2 \frac{\phi_{q,8}}{\epsilon_{\mathbf{q},8}} \left(\frac{1}{\omega + \epsilon_{\mathbf{q},8} + \epsilon_{\mathbf{k},8}} + \frac{1}{\omega + \epsilon_{\mathbf{q},8} - \epsilon_{\mathbf{k},8}} \right) \right] \end{aligned} \quad (59)$$

From now on we apply the approximation $\phi_{1,q} \approx 2\phi_{8,q} \approx 2\phi_q$ (cf. Ref.[26]) in the previous expressions. Doing so, for Eq. (57) we have

$$\delta\phi_k \simeq \frac{g^2}{24} \int_{\mu-\delta}^{\mu+\delta} dq \int_0^\infty d\omega [\mathcal{I}_t(\omega) - \mathcal{I}_{00}(\omega)] \frac{\phi_q}{\epsilon_{\mathbf{q}}} \left(\frac{1}{\omega + \epsilon_{\mathbf{q}} + \epsilon_{\mathbf{k}}} + \frac{1}{\omega + \epsilon_{\mathbf{q}} - \epsilon_{\mathbf{k}}} \right), \quad (60)$$

where we set $\epsilon_{\mathbf{q},8} = \epsilon_q$. In the rest of this paper we study the gluon self-energy effects on the value of only the octet gap function [Eq. (60)]. All the arguments presented in the next sections are valid for the singlet gap function as well [Eq. (59)].

Now, we have to know the values of the HDL and the CFL spectral densities for different values of the energy and the momentum. We discuss this in great detail in Sec. II C.

C. Dispersion Relations

Below the light cone, $p_0 < p$, the imaginary part of the HDL electric and magnetic gluons is nonzero, cf. Fig.(1) of Ref. [22]. Hence, the gluons are Landau damped. This is equivalent to regions Ia and IIa of Fig. (1) of this paper. In these regions, then, the spectral densities are calculated from Eq. (50). However, above the light cone, $p_0 > p$, regions IIb, IIc, and III, the imaginary parts are zero, and therefore, one has to find the spectral densities using Eq. (52). The dispersion relation of the electric and the magnetic gluons in this case are obtained from $p_0 = \omega_{00}(\mathbf{p})$ and $p_0 = \omega_t(\mathbf{p})$, respectively. The weak coupling limit, $m_g \ll \phi$, corresponds to region III of Fig. (1), where $\omega_{00}(0) = \omega_t(0) = m_g$.

In the CFL phase, below the light cone and for $p_0 < 2\phi$, region Ia of Fig. (1), the imaginary part of the electric and magnetic gluons are zero; one uses Eq. (52) to find the spectral densities. However, below the light cone and for $p_0 > 2\phi$, the imaginary parts are not zero. This is the case for $p_0 > p$ as well. In both of these cases, the spectral densities are obtained from Eq. (50), cf. Ref. [22]. Although the imaginary parts in regions IIb, $p < p_0 < E_{\mathbf{p}}^{88}$, and IIc, $E_{\mathbf{p}}^{88} < p_0 < E_{\mathbf{p}}^{18}$, are approximately at order of the HDL limit, they are very small in region III, $p_0 > E_{\mathbf{p}}^{18}$. In this region $\text{Im}\hat{\Pi}_{00}(p_0, \mathbf{p}) \sim \text{Im}\Pi_t(p_0, \mathbf{p}) \sim \phi^2$.

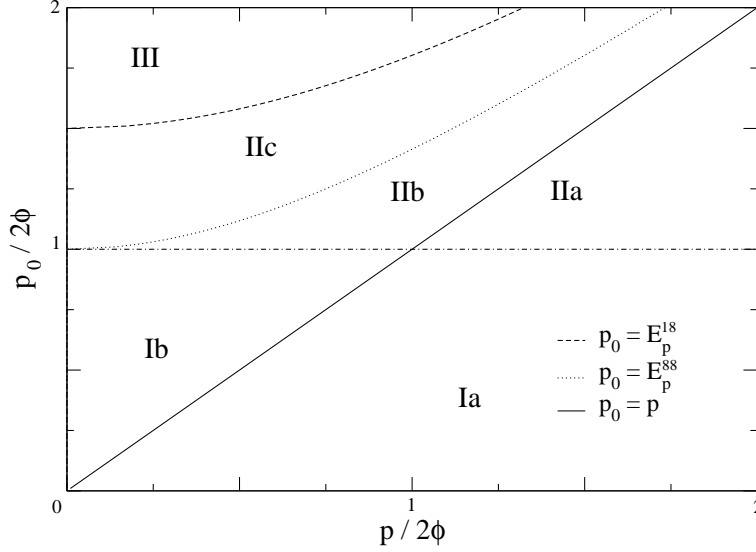


FIG. 1: Schematic plot of the (p_0, p) plane. In regions Ia and Ib, the HDL spectral densities are calculated from Eq. (50), and those in the rest of the plane, from Eq. (52). The CFL spectral densities are given by Eq. (50) in regions IIa, IIb, IIc, and III. In regions Ia and Ib, where the light plasmon mode exists, Eq. (52) is used to evaluate the spectral density. $E_p^{18} = \sqrt{p^2 + 9\phi_p^2}$ and $E_p^{88} = \sqrt{p^2 + 4\phi_p^2}$, cf. [22].

The Nambu-Goldstone (NG) bosons appear in region Ia, cf. Ref. [22]. The light plasmon modes, however, are present in regions Ia and Ib. In these regions, as stated before, the imaginary parts of the color-superconducting gluon self-energy are zero and the gluon energy and momentum are much smaller than the gap. The dispersion relation of the NG bosons is evaluated from $P_\mu P_\nu \Pi^{\mu\nu}(P) = 0$ and that for the light plasmons is calculated from Eq. (52). In this regime, $p_0, p \ll \phi$, the dispersion relation is approximately linear. Hence, we can expand the self-energies in terms of its momentum and energy. The leading terms are

$$\Pi^{00}(p_0, \mathbf{p}) \simeq \frac{m_g^2}{3} \left(2.6 + 0.4 \frac{p_0^2}{\phi^2} \right), \quad (61a)$$

$$\Pi^\ell(p_0, \mathbf{p}) \simeq \frac{m_g^2}{10} \frac{p_0^2}{\phi^2}, \quad (61b)$$

$$\Pi^{0i}(p_0, \mathbf{p}) \hat{p}^i \simeq -\frac{m_g^2}{150} \frac{p}{\phi} \frac{p_0^2}{\phi^2}, \quad (61c)$$

$$\Pi^t(p_0, \mathbf{p}) \simeq \frac{m_g^2}{20} \left(1 - 3 \frac{p_0^2}{\phi^2} \right), \quad (61d)$$

where the explicit forms of the self-energies are taken from Eq. (A2) of Ref. [22]. We refrain from presenting the details of the calculations leading to Eq. (61). Similar calculations for the 2SC phase are done in Ref. [27]. In this limit, the longitudinal self-energy $\hat{\Pi}^{00}(p_0, \mathbf{p})$ [cf. Eq. (4b) of Ref. [22]] is given by

$$\hat{\Pi}^{00}(p_0, \mathbf{p}) \simeq \frac{m_g^2}{10} \frac{p^2}{\phi^2} \left(1 + 0.2 \frac{p_0^2}{\phi^2} \right). \quad (62)$$

Having calculated the transverse self-energy in Eq. (61), the magnetic gluon propagator, $\Delta^t(P) = 1/(p_0^2 - p^2 - \Pi^t(P))$ becomes

$$[\Delta^t(P)]^{-1} \simeq \left[p_0^2 \left(1 + \frac{3}{20} \frac{m_g^2}{\phi^2} \right) - \mathbf{p}^2 - \frac{m_g^2}{20} \right]. \quad (63)$$

The light plasmon dispersion relation, therefore, reads

$$\omega_t \simeq \sqrt{\frac{20}{3}} \frac{\phi}{m_g} \sqrt{p^2 + \frac{m_g^2}{20}}. \quad (64)$$

Employing this and Eq. (63) in Eq. (53) one can find the associated residue

$$\mathcal{Z}_t(\mathbf{p}) \simeq \sqrt{\frac{5}{3}} \frac{\phi}{m_g p}. \quad (65)$$

We will make use of this result in Sec. III C.

In the next section we estimate the effect of the gluon self-energy on the value of the gap. For that, we have to investigate the spectral densities in the HDL limit as well as in the color-superconducting phase, cf. Equation (58).

III. RESULTS

In this section, I estimate the values of $\mathcal{I}_{00}(\omega)$ and $\mathcal{I}_t(\omega)$ to find the effects of the self-energy on the value of the gap.

A. Estimate for $\mathcal{I}_{00}(\omega)$

The qualitative results of this part are very similar to those of the 2SC phase for $\mathcal{I}_{00}^{11}(\omega)$ in Ref. [7], except that region IIc in Fig. (1) is new for the CFL phase. For $p_0 < 2\phi$, the spectral densities of the CFL phase are zero for both regions Ia and Ib, $\rho_{00} = 0$, cf. Fig.(3) of [22]. For the HDL limit, above the light cone, in region Ib, the spectral density is zero, therefore $\delta\rho_{00} = 0$. However, in region Ia, the HDL spectral density is regular and we have $\delta\rho_{00} = -\rho_{00}^{\text{HDL}}$. In this case, the spectral density is given by

$$\rho_{00}^{\text{HDL}}(\omega, p) = \theta(p - \omega) m_g^2 \frac{\omega}{p} \left\{ \left[p^2 + 2m_g^2 \left(1 - \frac{\omega}{2p} \ln \left| \frac{p + \omega}{\pi - \omega} \right| \right) \right]^2 + \left(\pi m_g^2 \frac{\omega}{p} \right)^2 \right\}^{-1}, \quad (66)$$

where m_g is the gluon mass at zero temperature, $m_g^2 = g^2 \mu^2 / 2\pi^2$. For p at order of ω , the HDL spectral density, Eq. (66), is approximately zero and for $p \gg \omega$ it is of order $\sim 1/p^5$. Therefore, using Eq. (58) we have

$$\mathcal{I}_{00}(\omega) \sim \frac{\omega}{m_g} \sim \frac{\phi}{m_g}. \quad (67)$$

Here, we neglected powers of g because they do not have any consequence on the order of $\mathcal{I}_{00}(\omega)$, cf.[7]. The effect of color superconductivity, however, appears for $p_0 > 2\phi$. In region IIa, both ρ_{00} and ρ_{00}^{HDL} are regular. Since for $\phi \rightarrow 0$ we recover the normal phase, $\delta\rho_{00} \rightarrow 0$ (cf. Ref. [22]), to leading we have

$$\int_{\omega}^{2\mu} dp p \delta\rho_{00}(\omega, p) \sim \frac{\phi}{m_g}. \quad (68)$$

On the other hand, in regions IIb and IIc, the HDL spectral densities are zero and ρ_{00} is of order $1/m_g^2$. Then, for region IIb

$$\int_{\sqrt{\omega^2 - 4\phi^2}}^{\omega} dp p \delta\rho_{00}(\omega, p) \sim \frac{\phi^2}{m_g^2}, \quad (69)$$

and for region IIc

$$\int_{\sqrt{\omega^2 - 9\phi^2}}^{\sqrt{\omega^2 - 4\phi^2}} dp p \delta\rho_{00}(\omega, p) \sim \frac{\phi^2}{m_g^2}. \quad (70)$$

In region III, the spectral density for the CFL phase is a smeared delta function and that for the HDL limit is a true one. Nevertheless, the integral over momentum in Eq. (58) makes $\mathcal{I}_{00}(\omega)$ regular. Qualitatively, the argument used for region IIa is valid at this region too. When $\phi \rightarrow 0$ we have $\rho_{00} \simeq \rho_{00}^{\text{HDL}}$, hence,

$$\int_0^{\sqrt{\omega^2 - 9\phi^2}} dp p \delta\rho_{00}(\omega, p) \sim \frac{\phi}{m_g}. \quad (71)$$

In conclusion, $\mathcal{I}_{00}(\omega)$ is at most of order ϕ/m_g .

B. Estimate for $\mathcal{I}_t(\omega)$

The arguments used in the previous section for regions where $p_0 > 2\phi$ are valid for $\mathcal{I}_t(\omega)$ too. However, for $p_0 < 2\phi$, the scenario is different because the light plasmon modes appear in the regions Ia and Ib. In these regions, since the imaginary parts of the gluon self-energy are zero, one has to use Eq. (52) to find the associated spectral densities. For both of these regions, the color-superconducting contribution is

$$\int_{\omega}^{2\mu} dp p \rho_t(\omega, p) = -p(\omega_t) \mathcal{Z}_t(p) \left(\left| \frac{\partial \omega_t(p)}{\partial p} \right| \right)_{p=p(\omega_t)}^{-1} \quad (72)$$

Making use of Eqs. (64) and (65) in Eq. (72) yields

$$\int_{\omega}^{2\mu} dp p \rho_t(\omega, p) \simeq -1. \quad (73)$$

In region Ia, one can approximate the HDL spectral density for $p \gg \omega$ as

$$\rho_t^{\text{HDL}}(\omega, p) \simeq \theta(p - \omega) m_g^2 \frac{\omega p}{p^6 + (\pi m_g^2 \omega)^2}. \quad (74)$$

This gives rise to

$$\int_{\omega}^{2\mu} dp p \rho_t^{\text{HDL}}(\omega, p) \simeq \frac{\phi}{m_g}. \quad (75)$$

Therefore in this region, to leading order, $\mathcal{I}_t(\omega)$ is at most of order one,

$$\mathcal{I}_t(\omega) = \int_{\omega}^{2\mu} dp p \delta \rho_t(\omega, p) \sim -1. \quad (76)$$

Furthermore, since in region Ib the HDL spectral density vanishes whereas the light plasmon spectral density persists, $\mathcal{I}_t(\omega)$ is of order ~ 1 too.

As mentioned, for the other regions, $p_0 > 2\phi$, $\mathcal{I}_t(\omega)$ has the same order of magnitude as $\mathcal{I}_{00}(\omega)$, namely, it is at most of order $\sim \phi/m_g$. In the next section, we take into account these effects on the value of the gap.

C. Estimate for $\delta\phi_k$

Having estimated the order of $\mathcal{I}_{00}(\omega)$ and $\mathcal{I}_t(\omega)$ we can now estimate the effect of the gluon self-energy on the gap. For $p_0 < 2\phi$, $\mathcal{I}_{00}(\omega)$ is at most of order $\sim \phi/m_g$ and $\mathcal{I}_t(\omega)$ of order ~ 1 . As mentioned earlier, we do not take into account the additional powers of the coupling g , because they are always accompanied by at least one power of the gap which is exponentially small in g , $\phi \sim \exp(-1/g)$. Employing Eq. (76) in Eq. (60), after performing the ω integral, we have

$$\delta\phi_k \sim -g^2 \int_{\mu-\delta}^{\mu+\delta} dq \frac{\phi_q}{\epsilon_{\mathbf{q}}} \ln \left| \frac{(2\phi + \epsilon_{\mathbf{q}})^2 - \epsilon_{\mathbf{k}}^2}{\epsilon_{\mathbf{q}}^2 - \epsilon_{\mathbf{k}}^2} \right|, \quad (77)$$

In Appendix B of Ref.[7] it is shown that this integral has a positive value and is at most of order $\sim \phi$. Therefore, $\delta\phi_k$ contributes to the $O(g)$ of the gap in Eq. (4), i.e. to the subsubleading order. Thus, the coefficients b and c are not modified. In addition, since $\delta\phi_k$ has a negative sign, the effect of the gluon self-energy is to decrease the value of the gap. This is in agreement with the previous result obtained using the Ginzburg-Landau theory, which shows near the critical temperature T_c the CFL gluon self-energy decreases the value of the gap [28].

IV. CONCLUSION

In conclusion, in the weak coupling limit, we studied the effect of CFL gluon self-energy on the solution of the gap. For the values of the energy and momentum, where the spectral densities of the CFL color superconductivity are regular, the effect of the gluon self-energy is very negligible, i.e. it does not even appear up to subsubleading order. However, for the energies and momenta, where the light plasmon mode appears, the effect is of subsubleading order.

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